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Quasi-normal Modes of AdS_5 Black Hole at $\mathcal{N} = 2$ Supergravity

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Abstract In this paper we consider the AdS_5 black hole at the $\mathcal{N} = 2$ supergravity background. By using the AdS/CFT correspondence we discuss about the quasi-normal modes of the scalar field in the black hole, which is dual of the scalar glueballs spectrum on the boundary. We obtain phase transition conditions from stable to unstable theory, which interpreted as confinement and deconfinement states in the QCD. We obtain the specific heat in terms of the temperature and charge of black hole, we find the temperature where the black hole is stable. Also we rewrite the equation of motion in the Schrödinger form and discuss the effective potential.

Keywords AdS/CFT correspondence · $\mathcal{N} = 2$ Supergravity · Black holes · Quasi-normal modes

1 Introduction

After 1997 the AdS/CFT correspondence has been introduced as a strong mathematical tools to study the gauge theories at the strong coupling [1–6]. The AdS/CFT correspondence relates a classical gravity on the AdS space to a gauge theory, with the conformal invariance, on the boundary of space-time. The main and known example of the AdS/CFT correspondence is the relation between the type IIB string theory in $AdS_5 \times S^5$ space and $\mathcal{N} = 4$ super Yang-Mills gauge theory on the 4-dimensional boundary of AdS_5 space. Recent studies in

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the case of reduction supersymmetry have shown that there is the relation between the string theory and the gauge theory with the less supersymmetry [7–14].

So, in this paper we are going to consider the AdS_5 black hole at the $\mathcal{N}=2$ supergravity background and calculate quasi-normal modes of the massless scalar field in the black hole. Recently it is found that $\mathcal{N}=2$ supergravity is an ideal laboratory [14–18], also discussion of the AdS/CFT correspondence with the $\mathcal{N}=2$ supergravity background have been studied [19–23]. The procedure of finding quasi-normal modes of the massless scalar field in the black hole performed by calculation of the retarded Green's function. It is known that the imaginary part of the Green's function is related to the quasi-normal modes. In order to obtain quasi-normal modes at various backgrounds there are many methods and previous studies [24–42]. Here, for the first time we consider the problem of the quasi-normal modes for the charged non-extremal black hole at the $\mathcal{N}=2$ supergravity background. In the recent work [42] the quasi-normal modes of the massless scalar field in the AdS black hole and scalar glueballs in a holographic AdS/QCD model at the finite temperature considered. This model known as the soft-wall model [43–45]. In Ref. [33] scalar glueballs spectrum at the finite temperature plasma has been studied by using AdS/QCD correspondence. Also the vector meson spectrum [43], $m^2=4c(n+1)$, scalar glueballs spectrum, $m^2=4c(n+2)$, and vector glueballs spectrum, $m^2=4c(n+3)$, have been discussed [44], where $n=0,1,2,\dots$ is the radial quantum number and the parameter $\sqrt{c}$ plays the role of a mass scale. The glueballs are described by the massless scalar field in the black hole. According to the Maldacena dictionary spectrum of scalar glueballs on the boundary of space-time are corresponding to the quasi-normal modes of the massless scalar fields in the black hole. In this formulation we deal with a five dimensional AdS space and a background dilaton field $\Phi(r)$. The AdS_5 line element in the Poincaré coordinates is given by,

$$ds^2_{AdS}=-dt^2+\frac{r^2}{L^2}\sum_{i=1}^{3}(dx^i)^2+dr^2, \tag{1}$$

where L denotes curvature radius of AdS space. In presence of background field $\Phi(r)$ in the bulk one needs to rewrite the action as the following,

$$S=-\frac{N_c^2}{4\pi^2}\int d^5x\sqrt{-g}e^{-\Phi}\mathcal{L}, \tag{2}$$

where the factor $e^{-\Phi}\mathcal{L}$ denotes the interaction of the dilaton field with some matter. The context of this paper summarized as the following.

In the next section we consider the AdS_5 black hole at the $\mathcal{N}=2$ supergravity theory in presence of the background dilaton field and try to obtain the black hole entropy and density of the specific heat. Then we extract conditions of stable/unstable transition of the theory which is corresponding to the confinement/deconfinement phase transition [46–49]. In the Sect. 3 we obtain the massless scalar field equation of motion and rewrite it in the form of the Schrödinger equation. In order to find quasi-normal modes we solve the Schrödinger like equation. Also we discuss about the effective potential in the Schrödinger like equation. Finally, by using the relation of retarded Green's function with the quasi-normal modes, in Sect. 4 we try to calculate the frequencies of the quasi-normal modes for the massless scalar field in the black hole. In Sect. 5 we summarized our results.

2 Thermodynamics of the AdS_5 Black Hole

We consider a soft-wall model with the background dilaton field $\Phi(r) = cr^2$ and the non-extremal black hole of $\mathcal{N} = 2$ AdS_5 supergravity,

$$
\begin{aligned}
ds^2 &= -\frac{f}{H^2}dt^2 + H\left(r^2 d\Omega_3^2 + \frac{dr^2}{f}\right),\\
f &= 1 - \frac{\eta}{r^2} + \Lambda^2 r^2 H^3, \\
H &= 1 + \frac{q}{r^2},
\end{aligned}
\tag{3}
$$

where Λ is the cosmological constant. q is called the black hole charge and it can be related to non-extremality parameter η by using the relation $q = \eta \sinh^2 \beta$ [11–13], where the β parameter is related to the electric charge of black hole. Notice that, in $q \to 0$ ($\eta \to 0$) limit, the line element in (4) is reduced to the line element of an extremal black hole (near extremal) with zero charge (infinitesimal charge). The coordinate r is axis along the black hole which defined in the interval $0 \le r \le \infty$. Let assume that the horizon of black hole is located at $r = r_h$. Also the boundary of space-time located at $r \to \infty$. There is the well known relation between Hawking temperature, horizon radius and charge of black hole as the following [50],

$$
T = \frac{2 + 3k - k^3}{2(1+k)^{\frac{3}{2}}} \frac{r_h}{\pi L^2},
\tag{4}
$$

where $k \equiv \frac{q}{r_h^2}$ and L denotes curvature radius of AdS space. Clearly, the $q \to 0$ ($\eta \to 0$) limit of (4) reduces to the Hawking temperature of $\mathcal{N} = 4$ super Yang-Mills theory in 4 dimensions, $T = \frac{r_h}{\pi L^2}$ [42]. Also one can see that the zero temperature obtained by taking $r_h^2 = \frac{q}{2}$ in (4). According to the Maldacena dictionary the temperature T is corresponding to the temperature of the field theory on the boundary. In the case of infinitesimal q and by eliminating $\mathcal{O}(q^6)$ one can write horizon radius as the following,

$$
r_h^2 = \frac{1}{\Lambda^2}(\pm\sqrt{1 - 3q^2\Lambda^4 + 6q\Lambda^2 + 4\eta\Lambda^2} - 1 - 3q\Lambda^2),
\tag{5}
$$

which reduces to the following relation for $q\Lambda \ll 1$,

$$
r_h^2 = -\frac{3}{4}q^2\Lambda^2 + \eta.
\tag{6}
$$

In that case one can find, $\sinh^4 \beta \le \frac{4}{3\eta\Lambda^2}$. Also in the case of $q = 0$ one can find $r_h = \frac{1}{\Lambda}$. Now by using the line elements (1) and (3), and integrating action one can obtain,

$$
S_{AdS} = \frac{N_c^2}{8\pi^2 c^2}\left[1 - c\left(r_h^2 + \frac{1}{c}\right)e^{-cr_h^2}\right],
\tag{7}
$$

$$
S_{BH} = S_{AdS} + \frac{N_c^2}{8\pi^2 c^2} qc[1 - e^{-cr_h^2}].
\tag{8}
$$

In the case of $q = 0$ there is no difference between the thermal AdS and the black hole AdS spaces. Therefore, phase transition at $q = 0$ limit happens at $T = \frac{r_h}{\pi L^2}$, so for $q < 0$ the thermal AdS space is dominant and for $q > 0$ the AdS black hole is dominant. Also there is another phase transition in the case of $q \neq 0$. By using (4) in the first order corrections of q one can write,

$$S_{BH} - S_{AdS} + \frac{N_c^2}{8\pi^2 c^2} qc\left[1 - e^{-\frac{1}{2}(\tilde{T}^2[1+\sqrt{1+\frac{6qc}{\tilde{T}^2}}]-3qc)}\right], \tag{9}$$

where we defined $\tilde{T} \equiv \sqrt{c}\pi L^2 T$. In this case the first order Hawking-Page phase transition temperature is $\tilde{T}_c^2 = \frac{3qc}{4}$. This phase transition in field theory on the boundary interpreted as confinement/deconfinement phase transition. It means that for $\tilde{T}_c^2 < \frac{3qc}{4}$ field theory is in the confinement phase and for $\tilde{T}_c^2 > \frac{3qc}{4}$ field theory is in the deconfinement phase. For the interesting case of $qc = 1$ the phase transition temperature obtained as $\tilde{T}_c^2 = 0.75$, so for $\tilde{T}_c^2 > 0.75$ dominant geometry is the AdS black hole which is the deconfinement phase and for $\tilde{T}_c^2 < 0.75$ dominant geometry is the five dimensional AdS space which is the confinement phase. The AdS black hole may be in stable or unstable phase. This fact understood by specifying the sign of specific heat. If the sign of specific heat becomes positive then the black hole is in the stable phase, in the other hand if the sign of specific heat becomes negative then the black hole is in the unstable phase. Therefore by using the relation,

$$\mathcal{C}_{BH} \equiv -\beta^2 \frac{\partial^2}{\partial\beta^2} S_{BH}, \tag{10}$$

in the first order approximation of q one can obtain,

$$\mathcal{C}_{BH} = \frac{N_c^2}{4\pi^2 c^2}\left[(3 + B - A)\left(\frac{qc}{4} - A\right) + 2(B - A)\right](A - B)e^{-\frac{1}{2}(A-3qc)}, \tag{11}$$

where

$$A \equiv \tilde{T}^2\left[1 + \sqrt{1 + \frac{6qc}{\tilde{T}^2}}\right] \quad \text{and} \quad B \equiv \frac{3qc}{\sqrt{1 + \frac{6qc}{\tilde{T}^2}}}.$$

In Fig. 1, we draw specific heat density in terms of $\tilde{T}$ for special case of $qc = 1$ and show that the specific heat changes the sign at the $\tilde{T}^2 \simeq 2$. It means that the stable/unstable phase transition of the charged non-extremal black hole happens at the temperature $\tilde{T}^2 \simeq 2$.

In the case of $q = 0$ one can find,

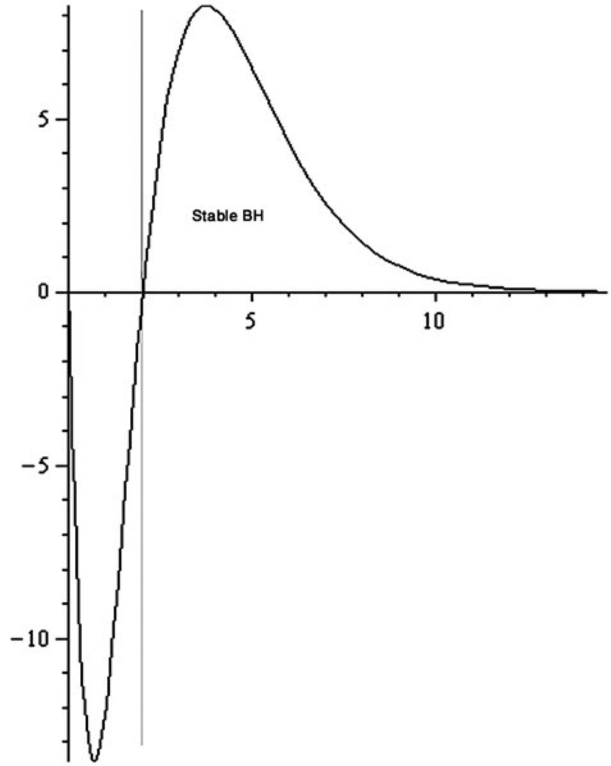
$$\mathcal{C}_{BH} = \frac{N_c^2}{4\pi^2 c^2}\tilde{T}^4[2\tilde{T}^2 - 5]e^{-\tilde{T}^2}. \tag{12}$$

From (12) one can see that the sign of specific heat is negative for $\tilde{T}^2 < 2.5$ and is positive for $\tilde{T}^2 > 2.5$. It means that for the $\tilde{T}^2 > 2.5$ there is the stable phase of the zero-charge extremal black hole. Therefore we conclude that the black hole charges decreases the phase transition temperature, so the charged black hole may be stable at lower temperature than the zero-charge black hole.

3 Equation of Motion

As we know from Maldacena dictionary, dual of the scalar glueball on the boundary of space-time is the massless scalar field in the bulk (black hole). In this section we want to

Fig. 1 Plot of the specific heat density versus the $\tilde{T}^2$ for the $qc = 1$ in units of $\frac{N_c^2}{4\pi^2 c^2}$



solve equation of motion for scalar field in the black hole and obtain quasi-normal modes. In order to obtain quasi-normal modes we try to rewrite the equation of motion in the form of Schrödinger equation. Then we find solutions of equation of motion by using asymptotic behavior of the Schrödinger like equation. Mentioned massless scalar field described by the following action,

$$S = -\frac{\pi^3 L^5}{4\kappa_{10}^2} \int d^5x \sqrt{-g} e^{-\Phi} g^{MN} \partial_M \phi \partial_N \phi, \tag{13}$$

where $\Phi(r) = cr^2$ is dilaton field and g_{MN} is the metric given by the relation (3). The parameters L and κ_{10} are the radius of the AdS space and the ten-dimensional gravity constant respectively. Indices M and N run from 0 to 4, so coordinates x^μ ($\mu = 0, 1, 2, 3$) is for four-dimensional boundary and $x^4 = r$ is extra coordinate along the black hole. By using the equation of motion for the scalar field ϕ in the action (13) one can obtain,

$$\frac{e^{\Phi}}{\sqrt{-g}} \partial_r (\sqrt{-g} e^{-\Phi} g^{rr} \partial_r \phi) + g^{\mu\nu} \partial_\mu \partial_\nu \phi = 0. \tag{14}$$

Now, by using the following Fourier transformation,

$$\phi(r, x) = \int \frac{d^4k}{(2\pi)^4} e^{ik.x} \bar{\phi}(r, k), \tag{15}$$

where $k_\mu = (-\omega, p_i)$, in the relation (14) one can write,

$$\frac{e^{cr^2}}{Hr^3}\partial_r\left(e^{-cr^2}\frac{H^2r^3}{f(r)}\partial_r\bar{\phi}\right) + \left(\frac{f(r)}{H^2}\omega^2 - Hr^2p^2\right)\bar{\phi} = 0, \tag{16}$$

where $p^2 = \sum_{i-1}^{3} p_i^2$. Now we introduce new coordinates $H^2\partial_r = f(r)\partial_{r_*}$, so we can integrate it and find explicit expression of r_* in terms of r,

$$r_* = (1+q\Lambda^2)r + \frac{\Lambda^2}{3}r^3 + \frac{(q+\eta)r}{2(q+r^2)} - \frac{(3q+\eta)}{2\sqrt{q}}\tan^{-1}\frac{r}{\sqrt{q}}, \tag{17}$$

which obviously reduces to $r_* = r + \frac{\Lambda^2}{3}r^3$ at $q = 0$ limit. Then we choose new variable as $\psi = e^{-\frac{D}{2}}\bar{\phi}$, where $D \equiv cr^2 - 3\ln r$. By using above definitions in (16) one can obtain following Schrödinger like equation,

$$\partial_{r_*}^2\psi + \omega^2\psi = V\psi, \tag{18}$$

where the effective potential defined as,

$$V = \frac{H^4}{f(r)}r^2p^2 - \frac{q}{r^2}\omega^2 - \frac{D''}{2} + \frac{D'^2}{4}, \tag{19}$$

where the prime denotes the derivative with respect ro the r_*. Asymptotical behavior of (18) is as the following,

$$\partial_r^2\psi = a(a-1)\frac{1}{r^2}\psi, \tag{20}$$

where there are two values for constant a which we named $a_\pm$ and given by,

$$a_\pm = \frac{1}{2}\left(1 \pm 2\sqrt{4 + q^3\Lambda^4\left(\frac{p^2}{\Lambda^2} - \omega^2\right)}\right). \tag{21}$$

Corresponding to the asymptotical behavior of the equation of motion of the scalar field one can write two solutions which satisfy the Schrödinger like equation,

$$\begin{aligned} \psi_1 &= r^{a_+}[1 + a_1r + a_2r^2 + a_3r^3 + a_4r^4 + \cdots] \\ \psi_2 &= r^{a_-}[1 + b_1r + b_2r^2 + b_3r^3 + \cdots] + b_4\psi_1\ln(cr^2). \end{aligned} \tag{22}$$

Only none-zero coefficients are a_2, a_4, b_2 and b_4. It is clear that $a_+ > 0$ and $a_- < 0$, therefore only the first term in ψ_2 will be remind at infinity. So one can interpreted $\psi_2 - b_4\psi_1\ln(cr^2)$ as a source of an operator on the boundary. In the other hand from [42] we know that it is glueball operator which is dual of the massless scalar field in to the bulk. In the case of $q = 0$ one can see that $a_+ = \frac{5}{2}$ and $a_- = -\frac{3}{2}$, so we recover solutions of Ref. [42],

$$\begin{aligned} \psi_1 &= r^{\frac{5}{2}}[1 + a_2r^2 + a_4r^4 + \cdots] \\ \psi_2 &= r^{-\frac{3}{2}}[1 + b_2r^2 + \cdots] + b_4\psi_1\ln(cr^2), \end{aligned} \tag{23}$$

where

$$a_2 = \frac{[(25+6c)\Lambda^2+\omega^2+4c][(3\omega^2+9+8c)\Lambda^2+\omega^2+4c-p^2-c^2]}{(35-9\Lambda^2)(2c\Lambda^2-p^2-c^2)}$$
$$-\frac{(3\Lambda^2\omega^2-2p^2-c^2)\Lambda^2}{2c\Lambda^2-p^2-c^2}$$
$$a_4 = -\frac{[(9+6c)\Lambda^2+\omega^2+4c]a_2+[(3\omega^2+2c)\Lambda^2-p^2-c^2]}{35-9\Lambda^2} \tag{24}$$
$$b_2 = -\frac{(3\omega^2+2c)\Lambda^2-p^2-c^2}{(1+6c)\Lambda^2+\omega^2+4c}$$
$$b_4 = \frac{(p^2+3\Lambda^2\omega^2)b_2+(2p^2-3\Lambda^2\omega^2)\Lambda^2}{8c}.$$

These coefficients are different with Ref. [42] because our system is different. We will use above solutions to construct retarded Green's function and extract quasi-normal modes.

It is known that the cosmological constant is infinitesimal parameter, so at the $q=0$ limit one can rewrite the effective potential (19) as the following,

$$V = (1-\Lambda^2r^2)r^2p^2 - \frac{(1-\Lambda^2r^2)^2}{2}\left(2c+\frac{3}{r^2}\right)$$
$$+\left(2c-\frac{3}{r^2}\right)(1-\Lambda^2r^2)\left[\frac{\Lambda^2}{4}r^2+\frac{c(1-\Lambda^2r^2)r^2}{2}-\frac{3}{4}-2\Lambda^2\right]. \tag{25}$$

One can see that at the $r\to r_h=\frac{1}{\Lambda}$ the effective potential vanishes. Therefore the Schrödinger like equation has the other solutions as incoming and outgoing wave functions which denoted by ψ_+ and ψ_-. These solutions obtained by using this fact that the effective potential vanishes at the horizon and the Schrödinger like equation take form of the free particle equation with solution of $\exp(\pm i\omega r_*)$. The negative sign interpreted as the incoming plane wave in to the horizon and the positive sign interpreted as the outgoing plane wave from the horizon. Therefore one can write the following expansion as the near horizon solution of the Schrödinger like equation,

$$\psi_\pm = e^{\pm i\omega r_*}[1+a_{1(\pm)}(1-\Lambda r)+a_{2(\pm)}(1-\Lambda r)^2+\cdots]. \tag{26}$$

The $\psi_\pm$ form the basis of any other wave functions, also it may be expand $\psi_\pm$ in terms of ψ_1 and ψ_2. Therefore one can relate solutions in (23) to (26) and vise versa,

$$\psi_\pm = \mathcal{A}_{(\pm)}\psi_2+\mathcal{B}_{(\pm)}\psi_1$$
$$\psi_{2,1} = \mathcal{C}_{(2,1)}\psi_-+\mathcal{D}_{(2,1)}\psi_+, \tag{27}$$

where $\mathcal{A}_{(+)}$, $\mathcal{A}_{(-)}$, $\mathcal{B}_{(+)}$, $\mathcal{B}_{(-)}$, $\mathcal{C}_{(2)}$, $\mathcal{C}_{(1)}$, $\mathcal{D}_{(2)}$ and $\mathcal{D}_{(1)}$ are ω and p dependent coefficients which determined by using boundary condition and related to each other by the following relation,

$$1 = \begin{pmatrix}\mathcal{A}_{(-)} & \mathcal{B}_{(-)}\\ \mathcal{A}_{(+)} & \mathcal{B}_{(+)}\end{pmatrix}\begin{pmatrix}\mathcal{C}_{(2)} & \mathcal{D}_{(2)}\\ \mathcal{C}_{(1)} & \mathcal{D}_{(1)}\end{pmatrix}. \tag{28}$$

This relation will be useful to find quasi-normal modes.

Before end of this section we would like to discuss about special behaviors of the effective potential. Here we set $p=0$, and rewrite the effective potential for $c=0$ and $q=0$ which is corresponding to the case of the extremal black hole without the dilaton field. In that case the effective potential given by,

$$V=\frac{3}{f(r)r^2}\left[\frac{3}{4}f(r)-\frac{1}{2f(r)}+(3-r^2)\Lambda^2\right], \tag{29}$$

where $f(r)=1+\Lambda^2r^2$. This potential behaves as the function ae^{-br^2} approximately, which is corresponding potential of the tachyon field. Therefore in the case of the extremal black hole with zero charge and without dilaton field background the scalar field of the theory is the tachyon field. It means that the scalar glueballs on the boundary of the space time are described by the tachyon field in the zero-charge extremal black hole.

4 Quasi-normal Modes

In order to obtain quasi-normal modes of scalar field ϕ in the black hole we should find retarded Green's function. As we know the imaginary part of the retarded Green's function is known as spectral function. It yields us to frequency of the quasi-normal modes. We use results of Ref. [42] to write the retarded Green's function. In that case by using equation of motion one can rewrite the action (13) as the following,

$$S=-\frac{\pi^3L^5}{4\kappa_{10}^2}\int d^4x dr\,\partial_r\left(\frac{H}{f(r)}\sqrt{-g}e^{-\Phi}\phi\partial_r\phi\right), \tag{30}$$

where $\phi_k(r)$ satisfy equation of motion and one can expand it in terms of obtained solution in the previous section. So, for the case of finite temperature one can write,

$$\phi_k(r)=e^{\frac{D}{2}}\left[\psi_2(r)+\frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}}\psi_1(r)\right]. \tag{31}$$

In the action (30) we should integrate over r and set $\phi(r_h)=0$. Then after some calculations it is found that the imaginary part of the retarded Green's function is proportional to the $\frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}}$. This quantity called spectral function. In the AdS/CFT point of view the spectral function is in the AdS side which is dual of the correlation function in the CFT side. Therefore we use following relation [42] to obtain imaginary part of the retarded Green's function,

$$\frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}}=\frac{\partial_r\psi_-\psi_2-\psi_-\partial_r\psi_2}{\partial_r\psi_-\psi_1-\psi_-\partial_r\psi_1}. \tag{32}$$

In the above expression we should evaluate functions in the near horizon limit. As we told already, for the case of $q=0$, near horizon solutions obtained at $r\approx\frac{1}{\Lambda}$. In that case one can obtain,

$$\mathrm{Im}\frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}}=\omega\frac{\psi_2\partial_r\psi_1-\psi_1\partial_r\psi_2}{(\omega\psi_1)^2+(\partial_r\psi_1)^2}, \tag{33}$$

where ψ_1 and ψ_2 are given by (23) at $r\to\frac{1}{\Lambda}$ limit. Then one can find the value of the expression (33) numerically.

There are many different ways to obtain the quasi-normal modes of the scalar field in the black holes. In this paper we would like to use the power series method [51] which is suitable for asymptotically AdS space-time. In that case we begin with the Schrödinger like equation (18). One can introduce new function such as $\psi=\varphi e^{-i\omega r_*}$ and put in (18), then one can obtain,

$$\partial_{r_*}^2\varphi-2i\omega\partial_{r_*}\varphi=V\varphi. \tag{34}$$

Now, by using the relation $\partial_{r_*}=\frac{H^2}{f(r)}\partial r$ we yield the following equation,

$$\frac{H^2}{f(r)}\frac{\partial^2\varphi}{\partial r^2}-\left[2i\omega+\frac{4qH}{f(r)r^3}+\frac{2H}{f(r)^2}\left(\frac{\eta}{r^3}+r\Lambda^2H^3-\frac{3q\Lambda^2}{r}H^2\right)\right]\frac{\partial\varphi}{\partial r}-\frac{f(r)}{H^2}V\varphi=0. \tag{35}$$

Equation (35) at the $q=0$ limit reduces to the following equation,

$$\frac{1}{f(r)}\frac{\partial^2\varphi}{\partial r^2}-\left[2i\omega+\frac{r\Lambda^2}{f(r)^2}\right]\frac{\partial\varphi}{\partial r}-f(r)V\varphi=0. \tag{36}$$

Because of a regular singularity at $r=r_h$ one can write a possible solution of (36) as,

$$\varphi_-(r)=\sum_m a_{m(-)}(1-\Lambda r)^m. \tag{37}$$

In order to find coefficients $a_{m(-)}$ we should put φ_- into (36), then by using the Dirichlet boundary condition we have,

$$\sum_m a_{m(-)}=0. \tag{38}$$

This leads us to obtain the roots of (38). This procedure can be performed numerically, the quasi-normal frequencies obtained.

5 Conclusion

In this paper we considered the AdS_5 black hole at the $\mathcal{N}=2$ supergravity theory. We used the AdS/CFT correspondence and discussed the scalar field in the AdS_5 black hole. At the first, we obtained the thermodynamics of the charged black hole. We found that the first order Hawking-Page phase transition happens at the temperature $\tilde{T}_c^2=0.75qc$, also we found stable/unstable phase transition of the charged non-extremal black hole at the temperature $\tilde{T}^2\simeq 2$. We have shown that this temperature for the extremal black hole with the zero charge increases to the temperature $\tilde{T}^2\simeq 2.5$. Therefore we found that the black hole charges reduces the temperature of the phase transition.

Then we obtained the equation of motion and write it in the form of the Schrödinger equation. We solved this equation and obtained the quasi-normal modes of the scalar field in the black hole. We discussed the effective potential and found that in the case of extremal black hole with zero charge and in the absence of dilaton field background the massless scalar field behaves such as tachyon field. Finally we calculated the imaginary part of the Green's function which used to obtain the quasi-normal frequencies numerically.

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